CHAPTER 13

EARTH PRESSURE

حيث ان ضغط السوائل متعادل فى جميع الاتجاهات فان ضغطى المياه البينية الحرة فى الاتجاهين الرأسى والافقى متساويين . هذا لا ينطبق على الضغوط الداخلية خلال التركيب الحبيبى للتربة (الضغوط الفعالة) اذ انها لا تكون متساوية . وللحصول على الضغط الرأسى الفعال على مستوى داخل التربة يتم تجميع حاصل ضرب سمك كل طبقة اعلى ذلك المستوى فى الكثافة الكلية او المغمورة , ويعتبر هذا الضغط جهدا رئيسيا . ولكن للحصول على الضغط الافقى الفعال للتربة , لتصميم حائط ساند مثلا , يلزم تعين معامل يضرب فى الضغط الرأسى الفعال للحصول على قيمة الضغط الافقى . هذا المعامل يعتمد على خشونة الحائط . ويلاحظ ان الضغط الافقى المتولد يعتمد بدرجة عالية على قيمة وشكل ازاحة الحائط .

In the design of retaining walls, sheet piles, or other earth retaining structures, it is necessary to compute the lateral pressure exerted by the retained mass of soil. A retaining structure is used for maintaining the ground surface at different elevations on either side of it. The soil retained is called backfill which may have horizontal or inclined top surface.

13.1 Plastic equilibrium in soil and coefficients of earth pressure

Analysis of stress conditions in a soil mass retained by a structure, or resisting the displacement of a structure against it, is based on the assumption that the soil is in plastic equilibrium, i.e. every point is in failure state. The Mohr-Coulomb equation represents such case as follows:

$$\frac{\sigma_1 - \sigma_3}{2} - \frac{\sigma_1 + \sigma_3}{2} \cdot \sin\phi = c \, . \cos\phi \qquad \text{.. .. .. (13.1)}$$

$$\text{or : } \sigma_1 = \sigma_3 . \tan^2(45 + \frac{\phi}{2}) + 2\, c \tan(45 + \frac{\phi}{2}) \qquad \text{.. .. .. (13.2)}$$

$$\text{Put : } N_\phi = \tan^2(45 + \frac{\phi}{2}) = \frac{1 - \sin\phi}{1 + \sin\phi} \qquad \text{.. .. .. (13.3)}$$

$$\therefore \sigma_1 = \sigma_3 \cdot N_\phi + 2\,c\,\sqrt{N_\phi} \qquad \text{.. .. .. (13.4)}$$

تعبر المعادلات السابقة عن العلاقة بين الضغط الرأسى والضغط الأفقى فى التربة عند وصولها لحالة الانهيار ، أو الأتزان اللدن. ويبين الشكل أسفله بيانى مور واتجاهات مستويات الانهيار المتولدة فى التربة فى حالتيها النشطة والسلبية.

ويلاحظ أنه فى حالة حدوث نقص فى الضغط الأفقى فى التربة – نتيجة تباعد حائط ساند عنها مثلا – تتحرك حبيبات التربة تلقائيا نحو الحائط محدثة للضغط الجانبى النشط. بينما اذا ضغط منشأ على التربة أفقيا – فى حالة كتلة معرضة لقوة أفقية مثبتة داخل التربة مثلا – فإن التربة تكون فى حالة إجهاد وانفعال سلبى ، أو فى حالة مقاومة ، وتجبر حبيبات التربة أن تتضاغط مولدة للضغط الجانبى السلبى – او المقاوم. وبديهى أن ضغط التربة السبيى يكون أعلى قيمة من الضغط النشط وهذا واضح ايضا من بيانى مور.

Equ. 13.4 can be presented by the Mohr circles, Fig. 13.1. In case of active earth pressure (ضغط التربة النشط), (σ_1) is the vertical stress due to the effective weight of soil plus any surcharge. At plastic equilibrium, (σ_3), which

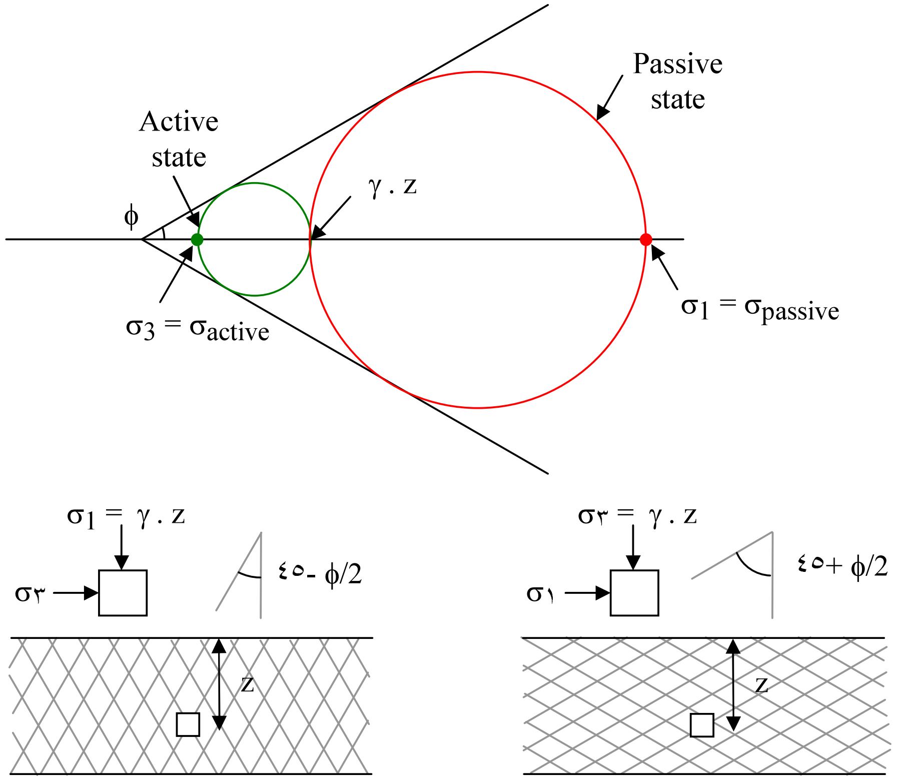


Fig. 13.1 Mohr circles for active and passive earth pressures

represents the lateral earth pressure, will be governed by the shear strength parameters of the soil as shown. The state of plastic equilibrium can be represented by the development of two sets of failure planes at an angle (45 - $\phi/2$) to the (σ_1) axis, i.e. to the vertical, representing possible failure planes.

In case of passive earth pressure (ضغط التربة السلبى), (σ_1) is the lateral earth pressure exerted against the soil. The effective weight of soil plus surcharge represent (σ_3). The state of plastic equilibrium can be represented by the development of two sets of failure planes at an angle (45 - ϕ /2) to the (σ_1) axis, i.e. at an angle (45 + $\phi/2$) to the vertical, representing possible failure planes.

In cohesionless soil, the ratio between the horizontal stress (σ_h) and the vertical stress (σ_v) is called the **coefficient of earth pressure**. When this soil is in the active state, the **coefficient of active earth pressure** (**k_a**) is defined by:

$$k_a = \frac{\sigma_h}{\sigma_v} = \frac{\sigma_3}{\sigma_1}$$

$$\therefore k_a = \frac{1}{\tan^2(45 + \frac{\phi}{2})} = \frac{1 - \sin\phi}{1 + \sin\phi} \qquad \text{.. .. .. (13.5)}$$

Therefore, the active earth pressure (p_a) can be generally expressed in the form:

$$p_a = \sigma_1 . k_a - 2c\sqrt{k_a} \qquad \text{.. .. .. (13.6)}$$

In case of the passive state, the coefficient of passive earth pressure (**k_p**) is defined by:

$$k_p = \frac{\sigma_h}{\sigma_v} = \frac{\sigma_1}{\sigma_3}$$

$$k_p = \tan^2(45 + \frac{\phi}{2}) = \frac{1 + \sin\phi}{1 - \sin\phi} \qquad \text{.. .. .. (13.7)}$$

Therefore, the passive earth pressure (p_p) can generally be expressed in the form :

$$p_p = \sigma_1 . k_p + 2 c \sqrt{k_p} \qquad \text{.. .. .. (13.8)}$$

Fig. 13.2 shows the active and passive states behind a structure. During the active state, the structure moves away from the backfill and a wedge breaks away from the rest of soil. This wedge is called failure wedge. In the passive state the structure moves towards the fill. The magnitude of the earth pressure depends on the movement of the structure.

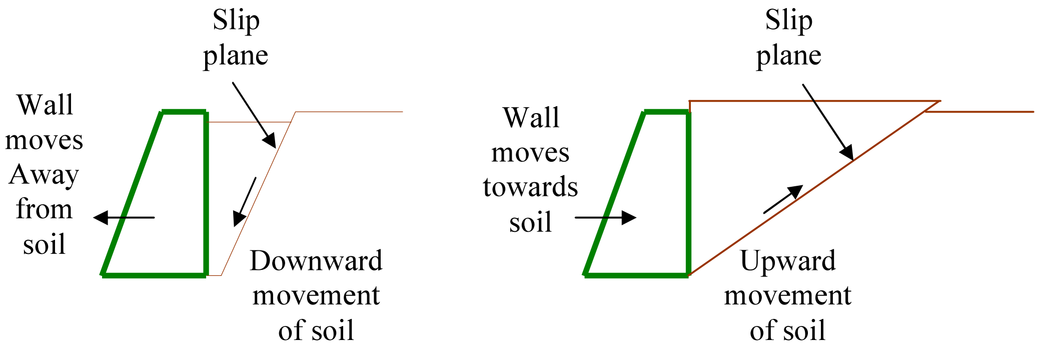


Fig. 13.2 Active and passive states in backfill

Fig. 13.3 shows that in dense sand very little displacement of nearly -0.5 % is sufficient to reach full active state. Much large displacement is required to reach passive state. A displacement of +0.5 % will cause development of only half the full passive pressure, and about 2 % displacement is required to reach full passive resistance. In loose sand such displacement may reach 15 %.

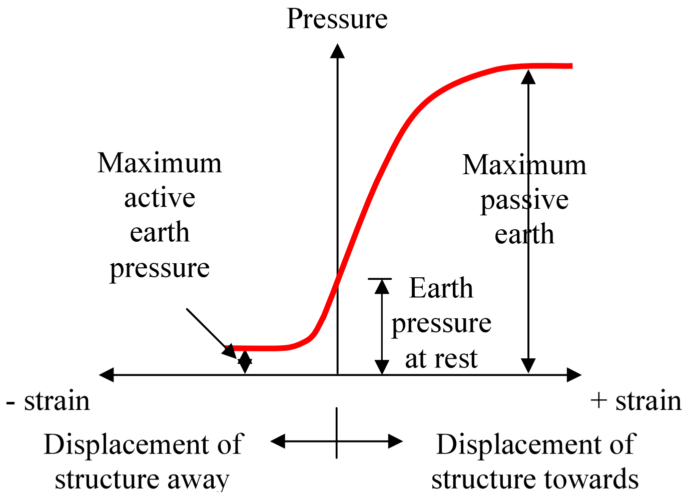


Fig. 13.3 Relation between structure displacement and lateral earth pressure

13.2 Earth pressure at rest

If no movement of wall occurs, for example in the case of unyielding rigid retaining wall, earth pressure at rest is to be considered. If the soil is assumed to be semi-infinite, homogeneous, elastic and isotropic, the ratio between the horizontal and vertical stresses, which is called coefficient of earth pressure at rest (k_o), is given by:

$$k_o = \frac{\nu}{1+\nu} \qquad \text{.. .. .. (13.9)}$$

where: ν = Poisson's ratio

The value of (k_o) for sands can be obtained from the following relation:

$$k_o = 1 - \sin\phi \qquad \text{.. .. .. (13.10)}$$

For clays, the value of (k_o) can be obtained from the effective angle of friction (ϕ') as follows :

$$k_o = 0.95 - \sin\phi' \qquad \text{.. .. .. (13.11)}$$

For normally consolidated clays:

$$k_o = 0.19 + 0.233 \log (PI\ \%) \qquad \text{.. .. .. (13.12)}$$

For over-consolidated clays values are obtained from Fig. 13.4:

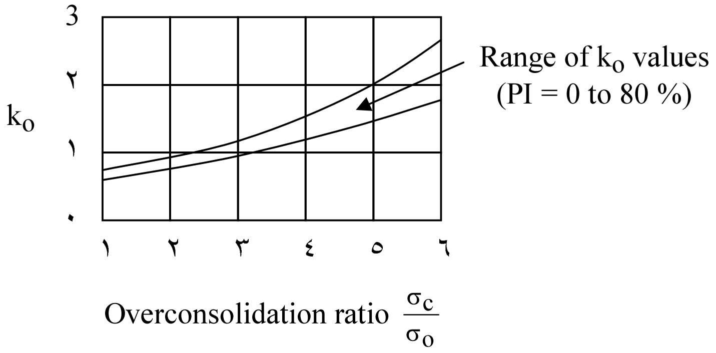


Fig. 13.4 Coefficient of earth pressure at rest
For overconsolidated clays

13.3 Rankine's theory of active earth pressure

نظرية رانكن لحساب الضغط الجانبى للتربة هى أكثر النظريات شيوعا واستخداما. وقد وضعت بفرض أن التربة ممتدة أفقيا إمتداد لانهائيا ومتجانسة وجافة وغير متماسكة , وأن سطح الأرض مستوى أفقيا أو مائلا , وأن الحائط يتحرك بمقدار يؤدى بالتربة الى حالة الاتزان اللدن وسطحه أملس. وقد إتسع استخدام هذه النظرية ليشمل حالات التربة المتماسكة والتربة غير المشبعة والمغمورة والمكونة من طبقات مختلفة الخواص , وكذلك تأثير وجود حمل على سطح الارض.

Rankine's theory was originally applied to uniform cohesionless soil. Later, it was extended to include cohesive soils, stratified and partially submerged soils. The following are the assumptions of Rankine 's theory :

(a) The soil is semi-infinite, homogeneous, dry and cohesionless,
(b) The ground surface is horizontal or sloping plane,
(c) The back of the wall is vertical and smooth,
(d) The wall displacement corresponds to deformation conditions causing plastic equilibrium in the soil .

13.3.1 Dry or wet backfill without surcharge

Consider an element at depth (z) below ground surface, Fig. 13.5. the horizontal pressure (P_{az}) will be the minimum principle stress (σ_3), and the vertical pressure will be the maximum principle stress (σ_1). Then:

$$p_{a_z} = k_a \,.\, \gamma \,.\, z$$

where : $\gamma =$ dry or bulk density of soil.

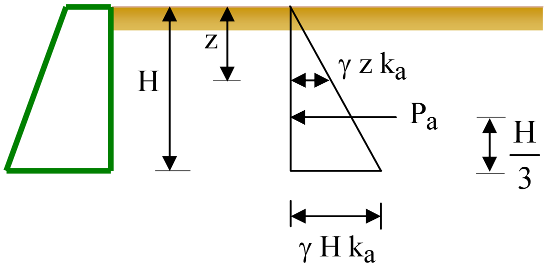


Fig. 13.٥ Active earth pressure for dry or wet backfill without surcharge

The earth pressure distribution is then linear. Its value at ground surface is zero and reaches maximum at the bottom and of the wall, i.e. at depth (H) :

$$p_{a_H} = k_a \cdot \gamma \cdot H$$

where: H = height of wall

The total active earth pressure (P_a), or the resultant lateral pressure per unit length of wall, acts at (H/3) above the base of wall and is given by:

$$p_a = \frac{1}{2} k_a \cdot \gamma \cdot H^2 \qquad \text{.. .. .. (13.13)}$$

13.3.2 Submerged backfill

The lateral pressure consists of two components, Fig. 13.6 (a) :

إذا كان منسوب المياه الجوفية مع سطح الارض , أى أن التربة مغمورة كلية بالمياه , فإن الضغط الجانبى سوف يكون محصلة ضغط التربة الرأسى الفعال (وهذا يعتمد على كل من الكثافة المغمورة ومعامل الضغط الجانبى للتربة) وضغط المياه الذى يعتمد على كثافة المياه . ويوضح الشكل توزيع ضغط التربة والمياه.

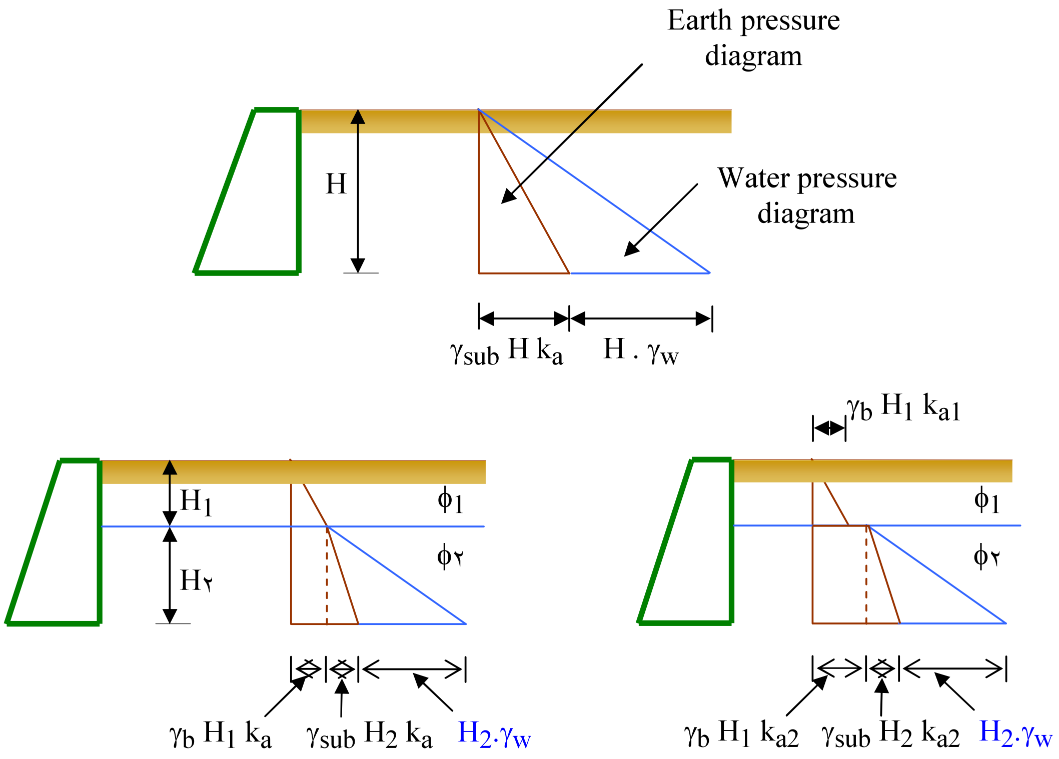
وفى حالة إذا كان منسوب المياه الجوفية على عمق معين أسفل سطح الارض فان ضغط التربة الجانبى سينقسم الى ثلاثة أجزاء, الجزء الاول الضغط المتسبب عن طبقة التربة الواقعة أعلى منسوب المياه الجوفية , والثانى الضغط المتسبب عن طبقة التربة الواقعة أعلى منسوب المياه الجوفية , والثالث الضغط المتسبب عن المياه الجوفية.

ويوضح الشكل أسفلة حالتين , الأولى شكل توزيع الضغط الجانبى على الحائط إذا كانت زاوية الاحتكاك للتربة أعلى وأسفل منسوب المياه الجوفية متساوية , والثانى اذا كانت زاوية الاحتكاك للتربة أعلى منسوب المياه الجوفية مختلفة عن تلك للتربة أسفل منسوب المياه الجوفية.

(a) lateral pressure due to the submerged weight of soil
(b) lateral pressure due to water

Thus, at depth (z) the lateral pressure is given by :

$$p_z = \gamma_{sub} \cdot z \cdot k_a + z \cdot \gamma_w$$



Case a: $\phi_1 = \phi_2$

Case b: $\phi_1 \neq \phi_2$

Fig. 13.٦ Lateral pressure for submerged backfill

If the backfill is partly submerged, the lateral pressure is given by:

$$p_z = \gamma_b \cdot H_1 \cdot k_a + \gamma_{sub} \cdot H_2 \cdot k_a + H_2 \cdot \gamma_w$$

It is to be noticed that this relation is only valid if (ϕ_1) is not changed below the water level. however, if such value is changed to (ϕ_2), the lateral pressure is given by :

$$p_z = \gamma_b \cdot H_1 \cdot k_{a1} + \gamma_{sub} \cdot H_2 \cdot k_{a2} + H_2 \cdot \gamma_w$$

where:

$$k_{a1} = \frac{1 - \sin \phi_1}{1 + \sin \phi_1} \qquad \text{and} \qquad k_{a2} = \frac{1 - \sin \phi_2}{1 + \sin \phi_2}$$

Notice that the earth pressure distribution will have a sudden change at the G.W.L. The value just above the G.W.L. depends of k_{a1}, whereas the value just below the G.W.L. depends of k_{a2}, the vertical effective stress for both cases being equal.

13.3.3 Backfill with uniform surcharge

يؤدى وجود حمل منتظم موزع على مساحة كبيرة على سطح الارض (نتيجة وجود افراد او معدات او حركة مرور .. الخ) الى زيادة قيمة الجهد الرأسى فى جميع المستويات داخل التربة بمقدار هذا الى زيادة ضغط التربة الافقى بقيمة ثابتة مع العمق .

The surface (q) will increase the vertical stress at any depth (z) to the value:

$$\sigma_1 = \gamma . z + q$$

The earth pressure will be as shown in Fig. 13.7 , its total at depth (z) will be given by :

$$p = k_a . q + k_a . \gamma . z \qquad \text{.. .. .. (13.14)}$$

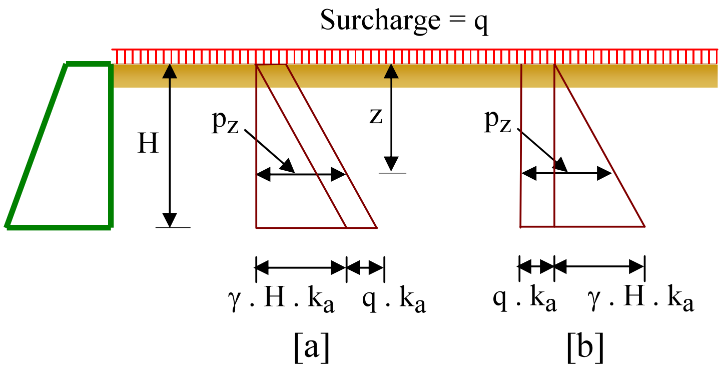


Fig. 13.٧ Lateral pressure for backfill with uniform surcharge
(The earth pressure diagram can be represented by either [a] or [b])

13.3.4 Backfill with sloping surface

Consider a soil element at depth (z), the top and bottom planes of the element are parallel to the surface of the backfill, sloping at an angle (β) to the horizontal, and its two sides are vertical. The conjugate stresses (σ) and (ρ) corresponding to these planes can be represented on the Mohr diagram as shown in Fig. 13.8. It can be proved that the earth pressure distribution on the wall as shown with (p_a) value at depth (z) is given by:

$$p_a = k_{a1} . \gamma . z$$

where :

$$k_{a1} = \cos\beta \cdot \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\beta}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \quad \text{.. .. .. (13.15)}$$

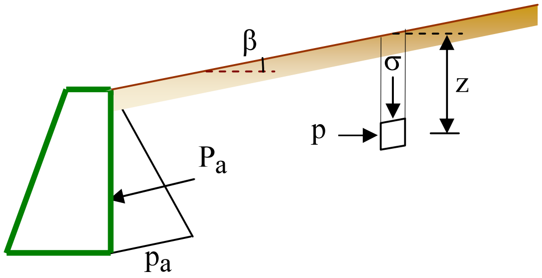


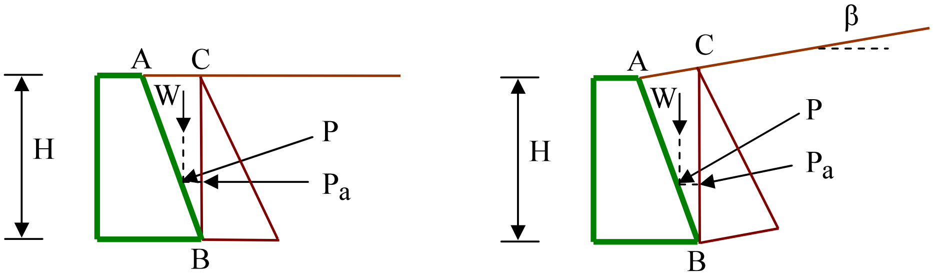
Fig. 13.8 Earth pressure for backfill with sloping surface

If the backfill is submerged, the lateral pressure due to the submerged weight of soil will act at angle (β) with the horizontal. The lateral pressure due to water will act normal to the wall surface.

13.3.5 Earth pressure on inclined wall

يحسب ضغط التربة الأفقى على حوائط المائلة , كما فى الأشكال أسفلة , بطريقة تقريبية. يتم أولا حساب الضغط الأفقى بفرض أن سطح الحائط رأسى يمر من أسفل بنهاية الحائط , ثم يحسب وزن التربة الواقع على السطح المائل للحائط , ويكون الضغط الكلى محصلة القيمتين.

Fig. 13.9 shows two case two cases of a retaining wall with inclined back supporting backfill. In the first case (a) the backfill has horizontal surface. The active pressure on a vertical plane (BC) is frist calculated. The total pressure is the resulatant of the horizontal pressure and the weight of the wedge (ABC).



[P is the resultant force for P_a and W and passes through their point of intersection]

Fig. 13.9 Earth pressure for backfill supported by inclined wall

If backfill has sloping surface, case (b). the same procedure is followed. The resultant pressure consists of the two compontents : the thrust which is being inclined at an angle (β), and the weight of wedge (ABC).

13.3.6 Active earth pressure for cohesive soil

بتطبيق المعادلة 13.6 للتربة المتماسكة (c # 0) نجد أن الضغط الأفقى للتربة عند سطح الارض – حيث يكون الضغط الرأسى الفعال مساويا للصفر – يكون سالبا. والمعنى الطبيعى لذلك أن التربة قادرة على الحفاظ على كيانها بدون سند او أى مساعدة خارجية , ولا تنهار التربة بالرغم من ظهور شروخ شد على سطحها. وتستمر هذه القدرة طالما ان الجمع الجبرى لحدى المعادلة سالبا. وبمعنى ابسط فإن التربة يمكن أن تحتفظ بحدودها رأسية بدون مساعدة حتى عمق (z_o) , يسمى عمق شووخ الشد , يتحدد بالمعادلة المبينة اسفلة.

It is shown in Equ. 13.6 that the earth pressure of cohesive soils increases with depth. The cohesion contributes constant reduction of a value of ($2\,c\sqrt{k_a}$). At ground surface, i.e. at z = 0, the lateral earth pressure is equal to ($-2\,c\sqrt{k_a}$). This pressure reaches zero at a depth (z_o) where :

$$z_o = \frac{2\,.\,c}{\gamma\,.\sqrt{k_a}} \qquad \text{.. .. .. (13.16)}$$

It follows that from ground surface to depth (z_o) the soil can stand vertical unsupported, i.e. along that depth no lateral pressure is exerted by the soil on the retaining wall. The total pressure is obviously zero at a depth ($2\,z_o$). The distribution is shown in Fig. 13.10.

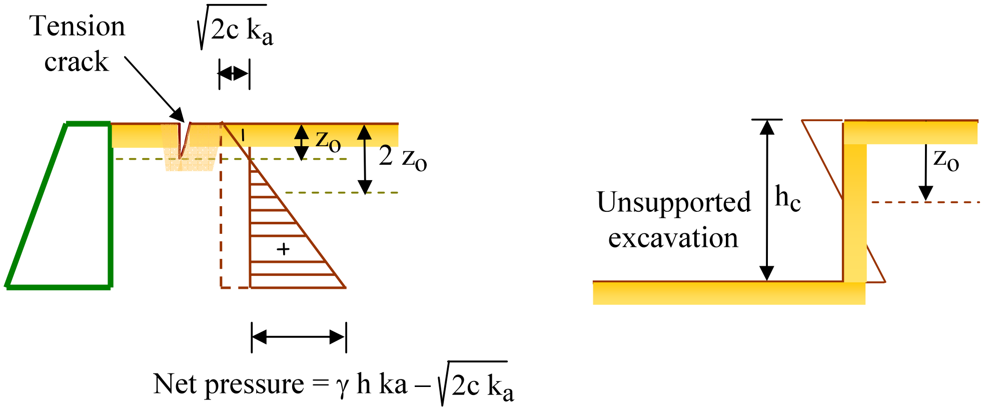


Fig. 13.10 Development of tension cracks in cohesive soil

The depth ($2\ z_o$) is called the critical height (h_c). This is the depth at which a cohesive soil should be able to stand with a vertical face unsupported. As crackes do occur, and soil does not necessarily remain adhered to the top portion of the wall up to height (z_o), it is usual to neglect the negative pressure and consider whole of the positive pressure below (z_o).

In case surcharge (q) acts on ground surface the lateral pressure (p_a) at any depth (z) is given by :

$$p_a = (\gamma \,.\, z + q)\,.\,k_a - 2\,.\,c\sqrt{k_a}$$

The depth of tension crack (zo) is given by :

$$z_o = \frac{2\,.\,c}{\gamma\,.\sqrt{k_a}} - \frac{q}{\gamma} \quad \text{.. .. .. (13.17)}$$

13.4 Passive earth pressure (Rankine 's theory)

Applying Equ. 13.8, the passive earth pressure (p_p) at depth (z) in cohesionless soil is given by :

$$p_p = k_p\,.\,\gamma\,.\,z$$

The total pressure on a wall with height (H) is given by :

$$p_p = \frac{1}{2} k_p\,.\,\gamma\,.\,H^2 \quad \text{.. .. .. (13.18)}$$

If a uniform surcharge of intensity (q) per unit area acts over the surface of backfill, the increase in the oassive pressure will be equal to ($k_p\ .\ q$). The total passive pressure at any depth (z) will be:

$$p_p = k_p\,.(\gamma\,.\,z + q)$$

If the top of the backfill is sloping at an angle (β) to the horizontal, the passive pressure will be given by :

$$p_p = \gamma\,.\,z\,.\cos\beta\,.\frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}$$

In cohesive soil the distribution of passive earth pressure is as shown in Fig. 13.11. it is noticed that at ground surface the passive pressure is given by ($2\,c.\sqrt{k_p}$).

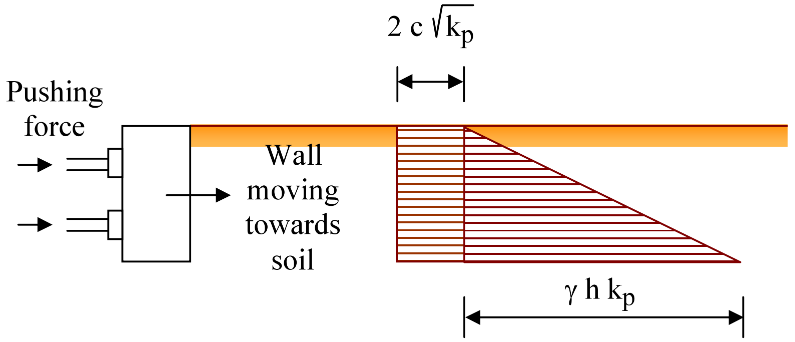


Fig. 13.11 Distribution of passive earth pressure in cohesive soil

13.5 Coulomb 's wedge theory

يتم تطبيق هذه النظرية اخذا فى الاعتبار خشونة الحائط السائد فى حالة ايجاد ضغط التربة النشط , او الخشونة المنشأ الضاغط فى حالة ايجاد ضغط التربة المقاوم . وتستخدم النظرية للتربة بانواعها المتماسكة وغير المتماسكة .

يعتمد الحل فى جميع الاحوال على فرض سطح انهيار للتربة , ثم تحليل اتزان كتلة التربة المحصورة بين الانهيار والمنشأ الساند او الضاغط زمنه حساب ضغط التربة . ويتم تكرار ذلك بفرض اسطح انهيار اخرى حتى الوصول الى اقصى او اقل قيمة للضغط فتكون هى ضغط التربة النشط او المقاومو على الترتيب . وتفترض سطوح الانهيار مستوية , ويكون ذلك مقاربا للواقع الا فى حالة ضغط التربة المقاوم على منشأ خشن (هذه الحالة غير موضحة هنا ويمكن الاطلاع فى المراجع العلمية) .

This theory takes into consideration the stability of the whole soil wedge between the failure plane and retaining wall. Friction and adhesion parameters (δ , c_w) between the wall and soil are taken into consideration. These parameters can be determined using the shear box, and normally defined as fractions of the shear strength parameters of the soil.

Due to consideration of the friction between soil and retaining wall, the rupture surface in both active and passive states is actually curved at its bottom. The theory, however, assumes that this surface is planer. The error involved in such approximation is small in the active state for all (δ)

values, and in the passive state for ($\delta < \phi/3$), otherwise the error is relatively large.

Fig 13.12 shows the method for determination of active and passive states for both cohesioless and cohesive soils. Several planes of rupture are chosen. For each plane the forces acting on the soil wedge are in equilibrium. The total forces acting on the wall due to earth pressure is determined from force polygons. By constructing the pressure envelope, the earth pressuer forces on the wall is determined. Notice that if the soil is conhesionless, each force poygon is reduced to a triangle of forces.

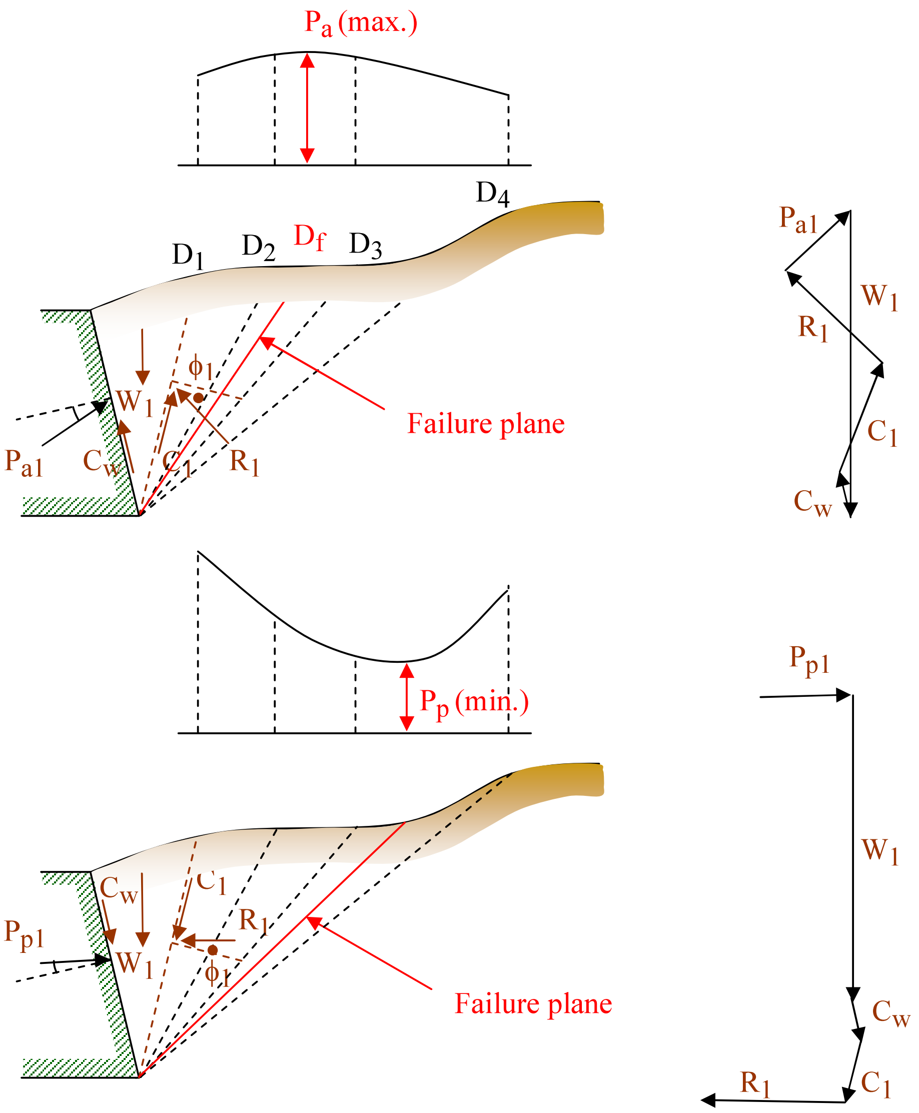


Fig.13.12 Coulomb's solution for active and passive earth pressures

Furthermore, the point of application of the maximum force due to earth pressure is determined following the constructions presented in Fig. 13.13. In case no external forces acts on soil surface, a line is draw parallel to the rupture plane meeting the wall face at the required point of application.

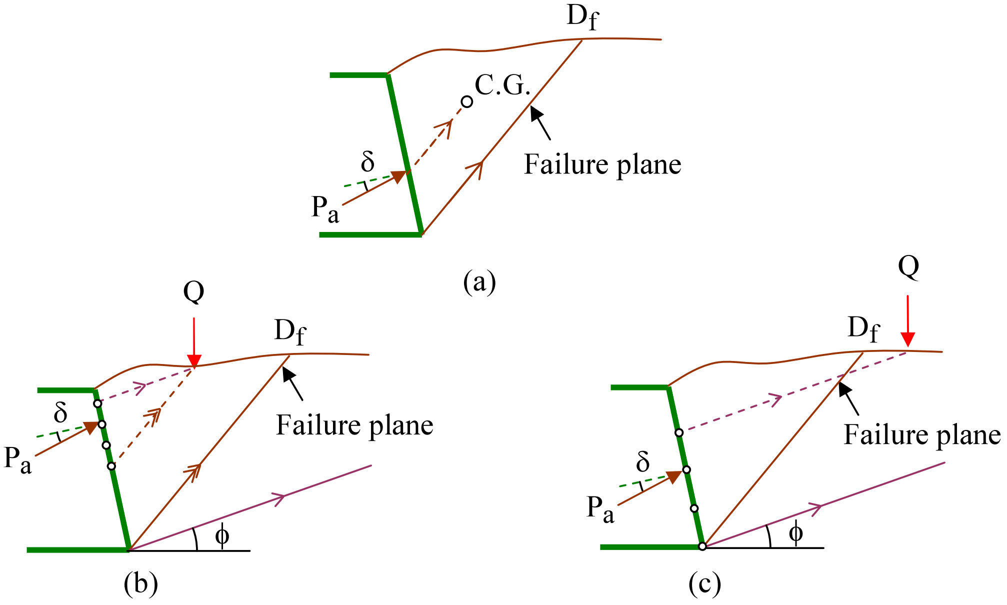


Fig. 13.13 Procedure for locating point of application of maximum force due to earth pressure on retaining wall

13.6 Rebhann's graphical method

Analysis solution using Coulomb's analysis for cohesionless backfill with planar surface proved that the maximum active pressure on the wall is obtained when triangles (ABC) and (BCE), Fig 13.14, have equal areas. This can be satisfied graphically following the Rebhann 's graphical method.

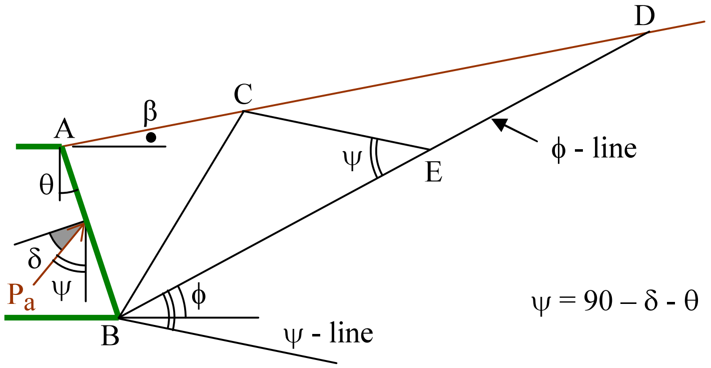


Fig. 13.14 Condition for maximum active earth pressure

The slip plane and the total active pressure according to Coulomb's wedge theory are obtained by the procedure shown in Fig. 13.15. this procedure is as follows :

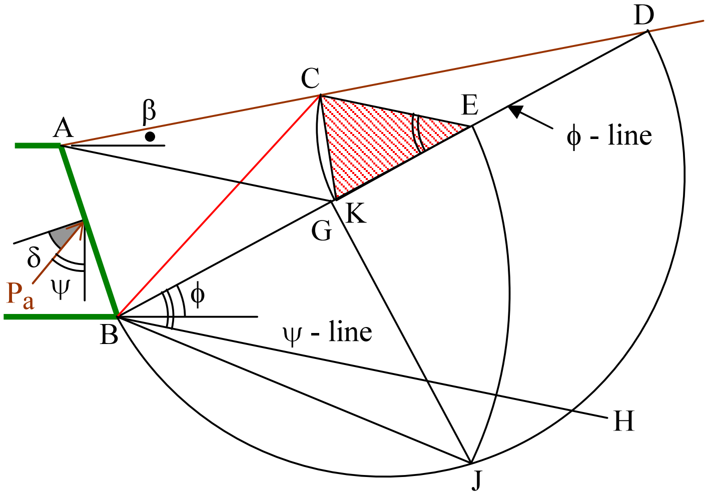


Fig. 13.1٥ Determination of maximum active pressure by Rebhann's method

(a) Draw the ground line and Φ-line to meet in point (D).
(b) Draw a semi-circle on (BD) as a diameter.
(c) through (B) draw a line (BH) at angle (ψ) with (BD), called the ψ-line through (A) draw line (AG) parallel to the ψ-line.
(d) Draw (GJ) perpendicular to BD) to meet the semicircle in (J).
(e) With (B) as center and (BJ) as radius, draw an are to cut (BD) in (E).
(f) Through (E) draw (Ec) parallel to the ψ-line. Line (BC) then presents the slip plane. With (E) as centre and (EC) as radius draw an sre to cut (BD) in (K). join (CK). Calculate the force due to active earth pressure on the wall from the following relation :

$$p_a = \gamma . (\text{Area of } \Delta \text{ KCE})$$

13.7 Culmann 's graphical method

تستخدم هذه الطريقة للتربة غير المتماسكة , ويفضل استخدامها لدراسة تأثير تغير مكان أحمال حيه على سطح الأرض على ضغط التربة الجانبى , كما هو موضح بالمثال رقم (٩) .

This method is based on Coulomb 's theory and used for cohesionless soils. It can be applied for irregular ground surface, for various types of surcharge and for layered soil having different densities. The procedure, Fig. 13.13, is as follows :

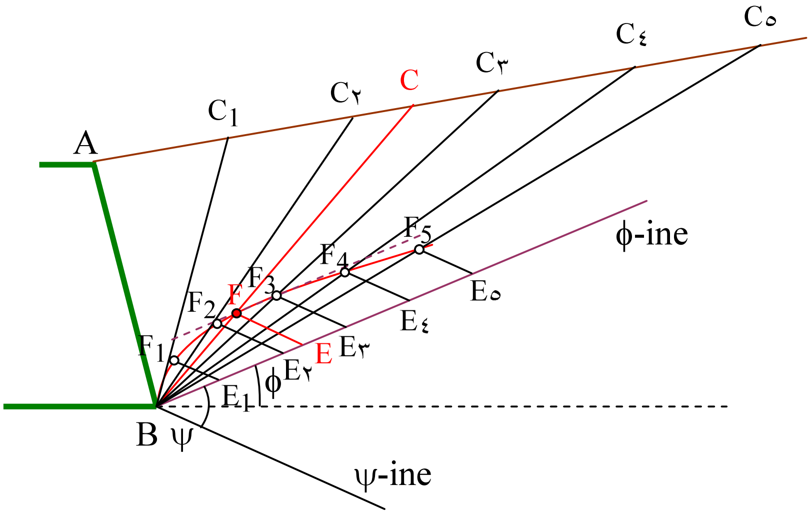


Fig. 13.16 Culmann's graphical method for active earth pressure

(a) Draw the ground line , Φ-line and ψ-line.
(b) Consider a slip plane (BC1). Calculate the weight of wedge (ABC1).
(c) Through (E1) draw (E1 F1) parallel to the ψ-line to cut the slip plane (BC1) in F1. (BE1 = W1 to scale)
(d) Repeat steps (b) and (c) to obtain several points (F2), (F3), (F4), .
(e) The line joining these points is called the Culmann 's line .
(f) Draw a tangent to the Culmann 's line (EF), and (BFC) represents the slip earth pressure is represented by the line (EF), and (BFC) represents the slip plane.
(g) To locate the point of application of the resultant pressure, refer to Fig. 13.13.

13.8 Design of gravity retaining wall

من اهم التطبيقات على حساب ضغط التربة الجانبى تصميم الحوائط السائدة . ونتناول هنا تصميم الحوائط التثاقلية التى يكون ثقلها مصدر اتزانها . ويمكن تطبيق اسلوب التحليل الموضح هنا لتصميم حوائط ذات اشكال اخرى او حوائط من الخرسانة المسلحة او اى منشأت اخرى ضاغطة على التربة .

A gravity retaining wall is that which resists which the lateral earth pressure by its own weight. In the design of retaining wall the following conditions should be satisfied :

(a) Maximum stress on soil under of retaining wall the following conditions should be satisfied :
(b) No tension stress will develop anywhere in the wall.
(c)The wall be safe against sliding.
(d) The wall be safe against overturning.

13.8.1 Check of stresses

الغرض من حساب الاجهادات على التربة هو مراجعة قيمتين , الأولى قيمة إجهاد الضغط القصوى , ويتم ذلك باعتبار الحمل الحى واقعا على سطح الارض بالكامل , والقيمة الثانية القيمة الأدنى للضغط والتى قد تنقلب الى شد مما يستدعى تعديل توزيع الاجهادات على التربة , أو يستدعى تغيير أبعاد الحائط . وفى الحالة الثانية يعتبر الحمل الحى واقعا حتى حدود الحائط (انظر Fig. 3.17) . وفى الحالتين يتم التحليل كما هو موضح اسفله.

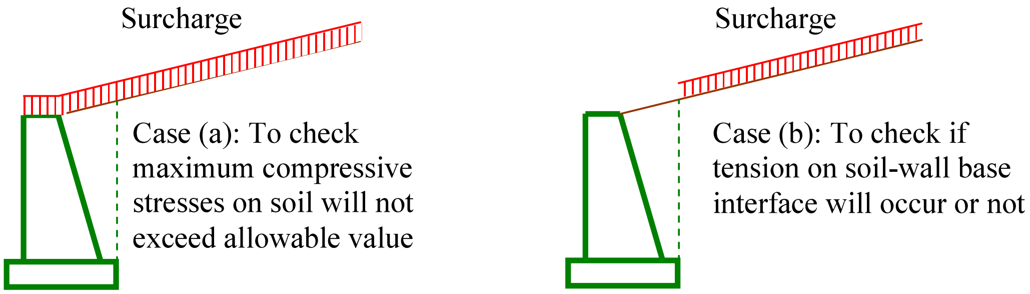

Surcharge
Case (a): To check maximum compressive stresses on soil will not exceed allowable value
Surcharge
Case (b): To check if tension on soil-wall base interface will occur or not

Fig. 13.17 Cases to be considered for design of gravity retaining wall

For the gravity wall shown in Fig. 13.18, let (P_a) the active earth pressure forces acting on plane (CD). Let (W_1) be the weight of soil above the wall, and let (W) be the weight of wall. To find the stress distribution on soil, the following procedure is commonly adopted:

(a) Find (N) : $N = W + W1 + P_{av}$
(d) Find moment (M_c) of all forces about point (C)
(c) Calculate (x)

$$x = \frac{\sum M_c}{N}$$

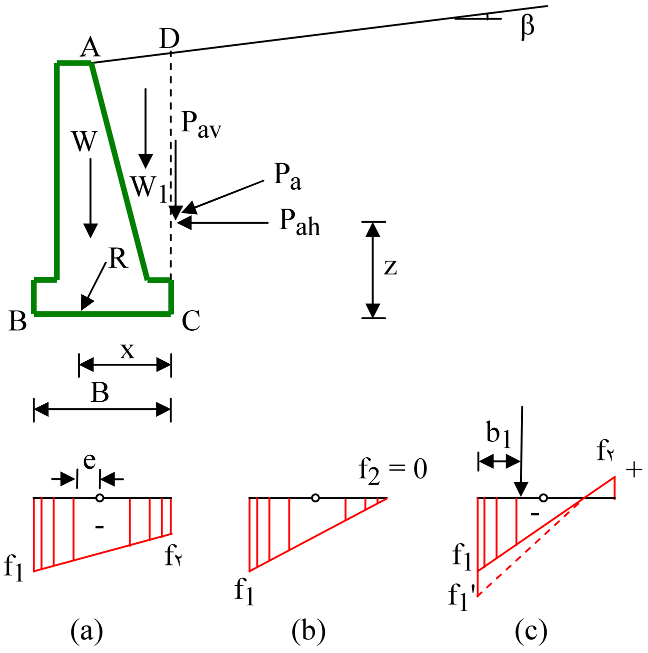


Fig. 13.18 Stresses on soil due to gravity retaining wall

The eccentricity (e) from the centre of base is given by :

$$e = \frac{B}{2} - x$$

The stresses the ends of the base are given by :

$$f_1 = \frac{-N}{B}\left[1 + \frac{6\,e}{B}\right]$$

$$f_2 = \frac{-N}{B}\left[1 - \frac{6\,e}{B}\right]$$

The value of (f_1) should not exceed (لا تتعدى) the total allowable soil stress. In case wind or seismic loads are taken into consideration, the allowable soil stress is increased by 15 % to 25 %.

To avoid tilting (ميل) of the wall, particularly in compressible soils, the following condition should be satisfied :

$$|f_1| \le 2\,|f_2|$$

In case of rock, or practically incompressible soil, (f2) may reach zero. Furthermore, if the resultant of forces cuts the base outside the middle third, tension on base occurs. However, this tension has no physical meaning, and in reality (فى الواقع) there will be no contact between the wall base and soil. Therefore, re-disstribution of the stresses will occur so that (f2) will be equal to zero and the maximum compression will modified as follows :

$$f_1' = \frac{2\ N}{3\ b_1}$$

Again, this stress distribution is only accepted for practically incompressible soil.

13.8.2 Check of sliding

يجب التأكد من ان مقاومة القص على سطح التماس بين قاعدة الحائط والتربة كافية لمقاومة القوى الافقية المؤثرة على الحائط بمعامل امان مناسب . فاذا لم يتوفر ذلك يجب زيادة عرض قاعدة الحائط او التصميم قاعدة بنتوء سفلى كما فى الشكل المبين لاستغلال ضغط التربة المقاوم فى توفير المقاومة المطلوبة .

The horizontal driving forces tending to cause sliding to the wall base is given by :

$$T = p_{a_h}$$

The resisting forces is :

$$S = c_w\ .\, B + (W + W_1).\tan\ \delta$$

$$\text{F.S. (sliding)} = \frac{S}{T}$$

where :

c_w = Adhesion between soil and wall base ($c_w \sim 2/3\ c_{soil}$)
δ = Angle of friction between soil and wall base ($\tan\ \delta \sim 2/3 \tan\ \phi$)
F.S. = Factor of safety

The factor of safety should not be (usually) less than 1.5.If this condition cannot be satisfied, either the wall base width will be increased, or a key, Fig. 13.19, will be used.

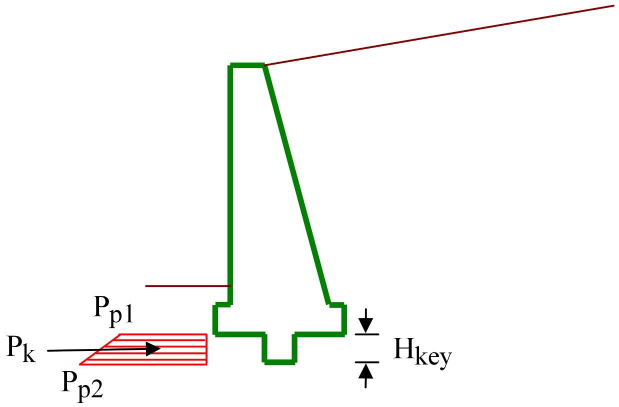


Fig. 13.19 Use of key to increase the factor of safety against sliding

The passive resistance generated along the key is calculated as follows :

$$p_{p_1} = \frac{N}{B} . k_p + 2\,c\;\; k_p$$

$$p_{p_2} = p_{p_1} + \gamma . k_p . H_{key}$$

Take $P_k = 0.5\ \Sigma\ Pp$ (assume half pressure develop for short displacement) Therefore, the safety will be accordingly (بالتبعية) increased.

13.8.3 Check of overturning

استكمالا لتصميم الحائط يجب التأكد من عدم تعرضه للانقلاب وذلك بحساب العزم الذى يساعد على الاتزان العزم الذى يؤدى الى الانقاب , والنسبة بينهما هى معامل الامان .

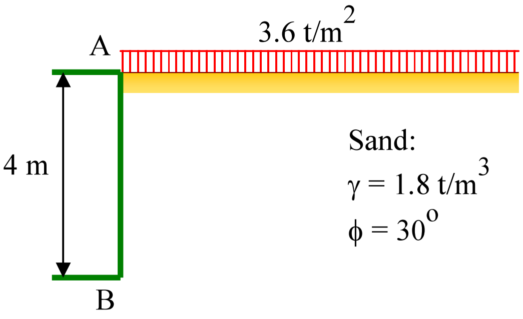
If the weight of wall is too small, overturning about the toe (point B, Fig. 13.18) may occur. The overturning moment is given by:

$$M_O = P_{ah} \cdot z$$

The stability moment is the moment of all permanent (دائم) loads about point (B), Fig. 13.17. The factor of safety against overturning is obtained by dividing the stability moment by the overturning moment. Its value should not be (usually) less than 1.5. If this condition cannot be satisfied, the wall base width is to be increased.

13.9 Examples

(1) For the shown wall with smooth vertical back and sand backfill, determine the magnitude and point of application of the active earth pressure per meter length of the wall.



Solution:

$$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin 30}{1+\sin 30} = 0.333$$

At any point at depth z, apply Equ. (13.14):

$$p_a = q.k_a + \gamma.z.k_a$$

$$p_{a(A)} = 3.6 \times 0.333 = 1.2 \text{ t/m}^2$$

$$p_{a(B)} = 1.2 + 1.8 \times 4 \times 0.333 = 1.2 + 2.4 = 3.6 \text{ t/m}^2$$

$$P_1 = 1.2 \times 4 = 4.8 \text{ t/m'}$$

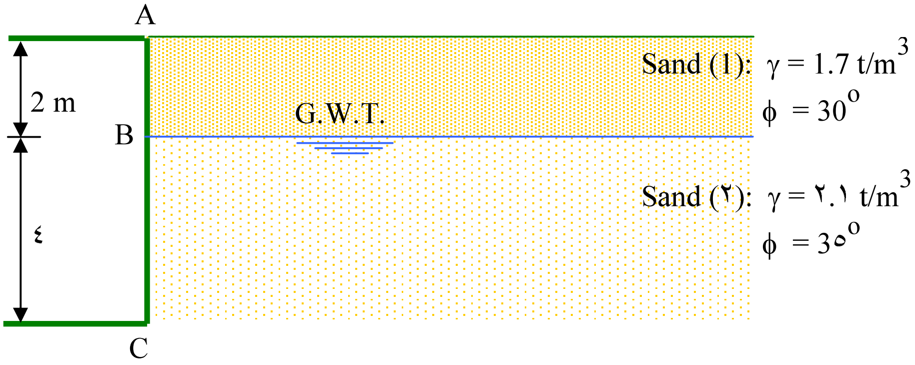
$$P_2 = \frac{1}{2} \times 2.4 \times 4 = 4.8 \text{ t/m'}$$

$$\therefore P_{total} = 4.8 + 4.8 = 9.6 \text{ t/m'}$$

Acting at :

$$y = \frac{(4.8 \times \frac{4}{2} + 4.8 \times \frac{4}{3})}{9.6} = 1.67 \text{ m}$$

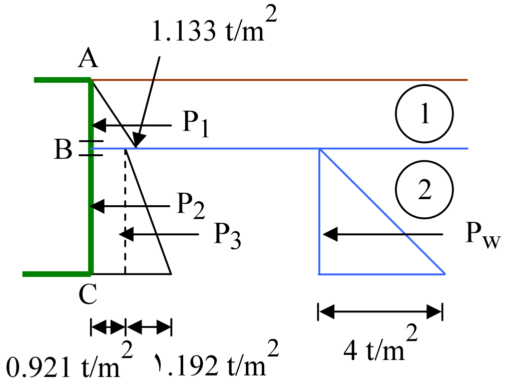
(2) Draw the active earth pressure and water pressure distributions on the shown vertical wall with smooth back. Calculate the lateral forces acting on the wall.



Solution:

$$k_{a1} = \frac{1-\sin 30}{1+\sin 30} = 0.333$$

$$k_{a2} = \frac{1-\sin 35}{1+\sin 35} = 0.271$$



$p_{aA} = 0$

p_{aB} (at end of layer 1) = $1.7 \times 2 \times 0.333 = 1.333\ t/m^2$

p_{aB} (at top of layer 2) = $1.7 \times 2 \times 0.271 = 0.921\ t/m^2$

$p_{aC} = 0.921 + (2.1-1) \times 4 \times 0.271 = 2.113\ t/m^2$

$p_w = 4\ t/m^2$

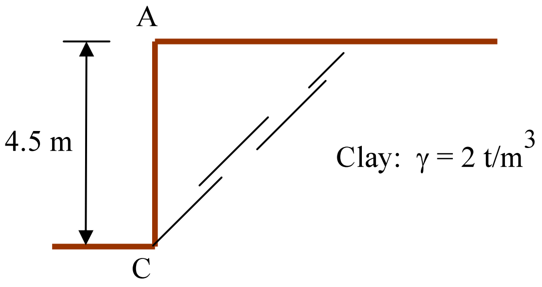
$P_1 = 0.5 \times 1.133 \times 2 = 1.133\ t/m'$

$P_2 = 0.921 \times 4 = 3.69\ t/m'$

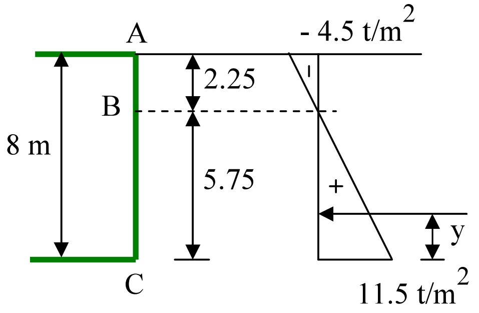
$P_3 = 0.5 \times 1.192 \times 4 = 2.39\ t/m'$

$P_w = 0.5 \times 4 \times 4 = 8\ t/m'$

(3) A vertical cut in a clay collapsed when the depth of excavation (حفر) reached 4.5 m. Calculate the clay cohesion. If a vertical cutting in this clay, 8 m deep, is supported by a retaining wall, calculate the earth pressure force acting on the wall.



Solution:



$$h_c = \frac{4\,c}{\gamma\sqrt{k_a}} = 4.5$$

For $\phi = 0, k_a = 1$

$$\therefore c = \frac{4.5 \times 2}{4} = 2.25 \text{ t/m}^2$$

$$p_a = (\gamma\, z + q) \cdot k_a - 2\,c\sqrt{k_a}$$

$$p_{aA} = -2\,c\sqrt{k_a} = -2 \times 2.25\sqrt{1} = -4.5 \text{ t/m}^2$$

$$p_{aC} = \gamma\, z \cdot k_a - 2\,c\sqrt{k_a}$$

$$= 2 \times 8 \times 1 - 2 \times 2.25\sqrt{1} = 11.5 \text{ t/m}^2$$

To find distance AB :

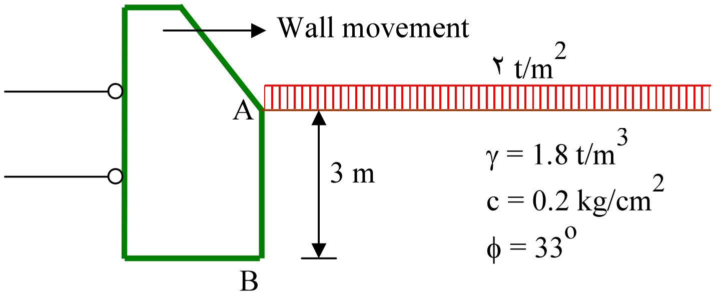
$$AB = \frac{4.5}{11.5 + 4.5} \times 8 = 2.25 \text{ m}$$

$$\therefore BC = 8 - 2.25 = 5.75 \text{ m}$$

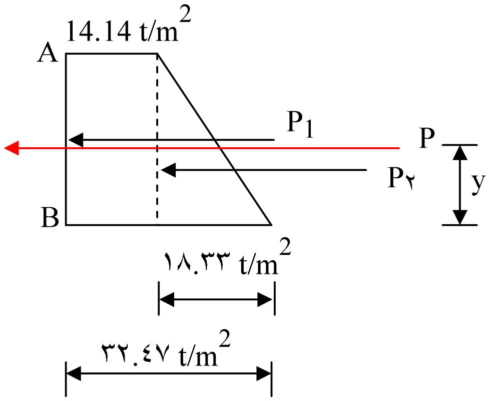
$$P = 0.5 \times 5.75 \times 11.5 = 33.06 \text{ t/m'}$$

$$y = \frac{5.75}{3} = 1.92 \text{ m}$$

(4) For the shown wall with smooth vertical face, calculate the total passive earth pressure on the side AB.



Solution:



$$p_p = (\gamma\, h + q)\, k_p + 2\, c \sqrt{k_p}$$

$$k_p = \frac{1 + \sin 33}{1 - \sin 33} = 3.39$$

$$p_{pA} = 2 \times 3.39 + 2 \times 2 \times \sqrt{3.39} = 14.14 \text{ t/m}^2$$

$$p_{pB} = (2 + 1.8 \times 3) \times 3.39 + 2 \times 2 \times \sqrt{3.39} = 32.47 \text{ t/m}^2$$

$$P_1 = 14.14 \times 3 = 42.42 \text{ t/m'}$$

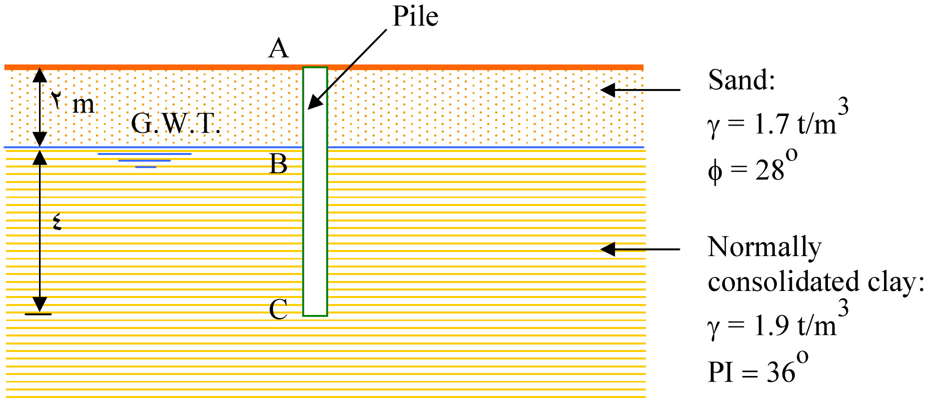
$$P_2 = 0.5 \times 18.33 \times 3 = 27.47 \text{ t/m'}$$

$$\therefore P_{total} = 42.42 + 27.47 = 69.89 \text{ t/m'}$$

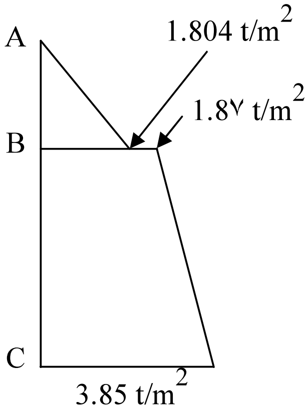
Acting at :

$$y = 1.67 \text{ m}$$

(5) Draw the lateral earth pressure distribution on the shown driven pile (خازوق بالدق).



Solution:



For sand:

$k_o = 1 - \sin\phi = 1 - \sin 28 = 0.53$

$p_{oA} = 0$

p_{oB} (lower end of sand layer)

$= 2 \times 1.7 \times 0.53 = 1.804\ t/m^2$

For clay:

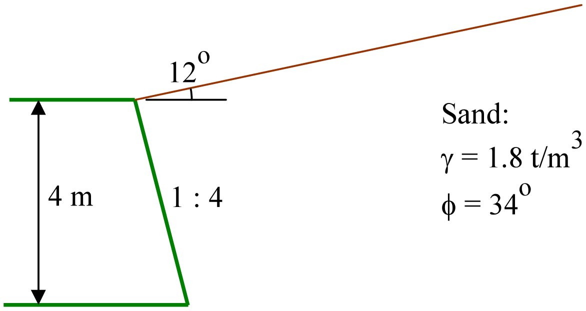
$k_{\,o} = 0.19 + 0.233 \log PI$

$\quad = 0.19 + 0.233 \log 36 = 0.55$

p_{oB} (top of clay layer) $= 2 \times 1.7 \times 0.55 = 1.87\ t/m^2$

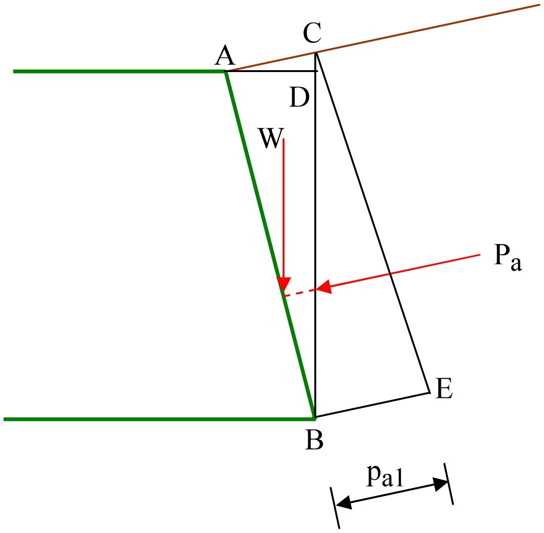
$p_{oC} = 1.87 + 0.9 \times 4 \times 0.55 = 3.85\ t/m^2$

(6) Calculate the total active earth pressure on the shown inclined wall with sloping backfill. Use Rankine's method.



Sand:
$\gamma = 1.8$ t/m^3
$\phi = 34^o$

Solution:



Apply Eq.13.15 :

$$k_{a1} = \cos\beta . \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

$$k_{a1} = \cos 12 . \frac{\cos 12 - \sqrt{\cos^2 12 - \cos^2 34}}{\cos 12 + \sqrt{\cos^2 12 - \cos^2 34}} = 0.3$$

$AD = 1\,\mathrm{m}$

$CD = 1 \times \tan 12 = 0.213\,\mathrm{m}$

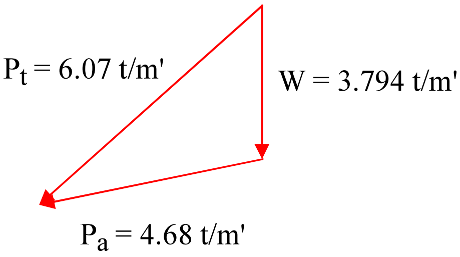
$CB = 4 + 0.213 = 4.213\,\mathrm{m}$

$p_{a1} = 1.8 \times 4.213 \times 0.3 = 2.27$ t/m'

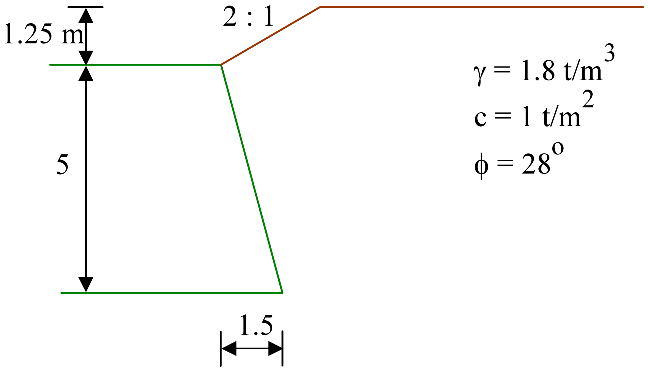
$W = 1.8 \times 0.5 \times 4.213 \times 1 = 3.794$ t/m'

From graph :

$P_t = 6.07$ t/m'



(7) Use coulomb's solution to calculate the active earth pressure on the shown wall. Take $\delta = 2/3\ \phi$, $c_w = c/2$. Consider development of tension cracks.



Solution:

$$k_a = \frac{1 - \sin\phi}{1 = \sin\phi} = \frac{1 - \sin 28}{1 = \sin 28} = 0.36$$

$$z_o = \frac{2\,c}{\gamma\sqrt{k_a}} = \frac{2 \times 1}{1.9\sqrt{0.36}} = 1.75\ \text{m}$$

From figure:

$bd = 3.6\ \text{m} \quad \therefore C_w = (0.5 \times 1) \times 3.6 = 1.8$ t/m'

$W_{a\text{-}a1\text{-}b1\text{-}b} = 1.75 \times 2.2 \times 1.9 = 7.32$ t/m'

$W_{b\text{-}d\text{-}b1} = 0.5 \times 2.2 \times 3.6 \times 1.9 = 7.52$ t/m

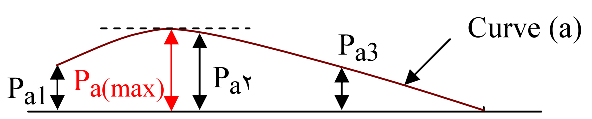
Sum $= 14.84$ t/m'

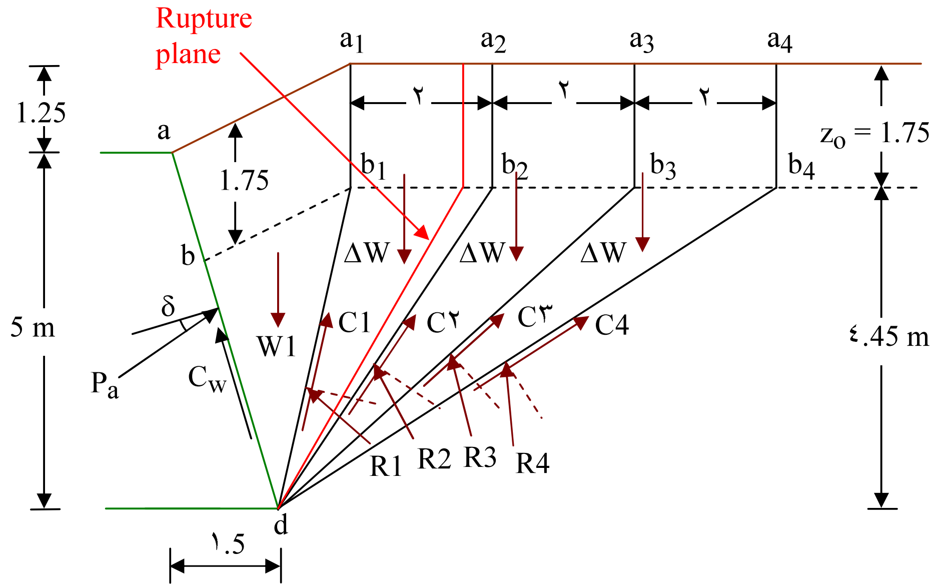
$W_{a1\text{-}a2\text{-}b2\text{-}b1} = 2 \times 1.75 \times 1.9 = 6.65$ t/m

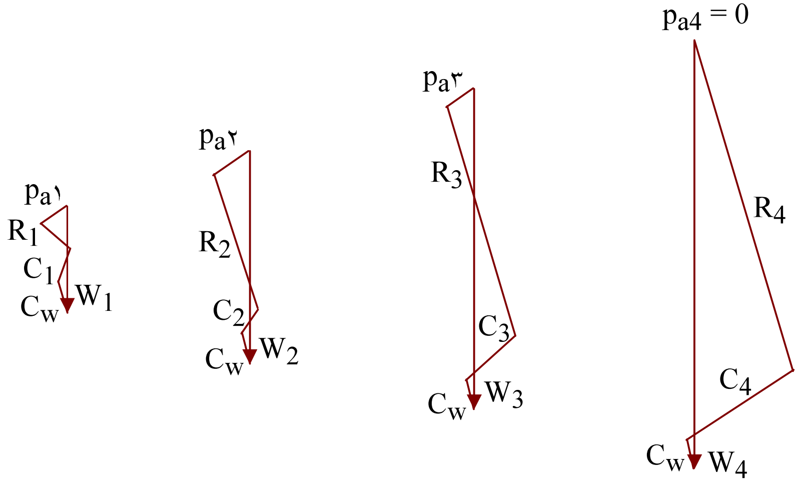
$W_{d\text{-}b1\text{-}b2} = 0.5 \times 2 \times 4.45 \times 1.9 = 8.64$ t/m

Sum $= 15.11$ t/m'

For each trial: $C = L \times c = L \times 1, \quad \delta = \frac{2}{3} \times 28 = 18.7^o$





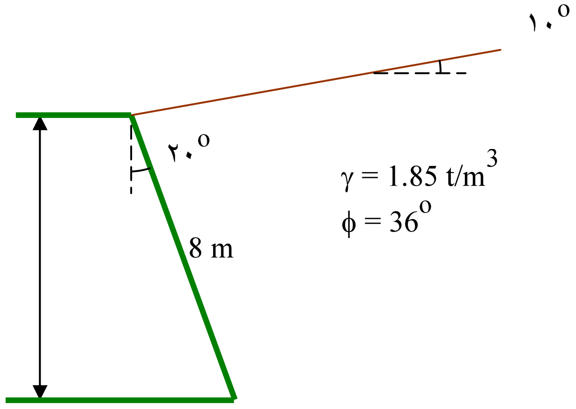


From figures:

Trial No.	W t/m	ΔW t/m	W(total) t/m	L M	C t/m	Pa t/m
1	14.84	-	14.84	4.5	4.5	6.4
2		15.11	29.95	5.3	5.3	10.8
3		15.11	45.06	6.7	6.7	6.0
4		15.11	60.17	8.3	8.3	0

From curve (a): $P_{a\ (max.)} = 11$ t/m'

(8) Use Rebhann's graphical method to calculate the active earth pressure for the shown wall.

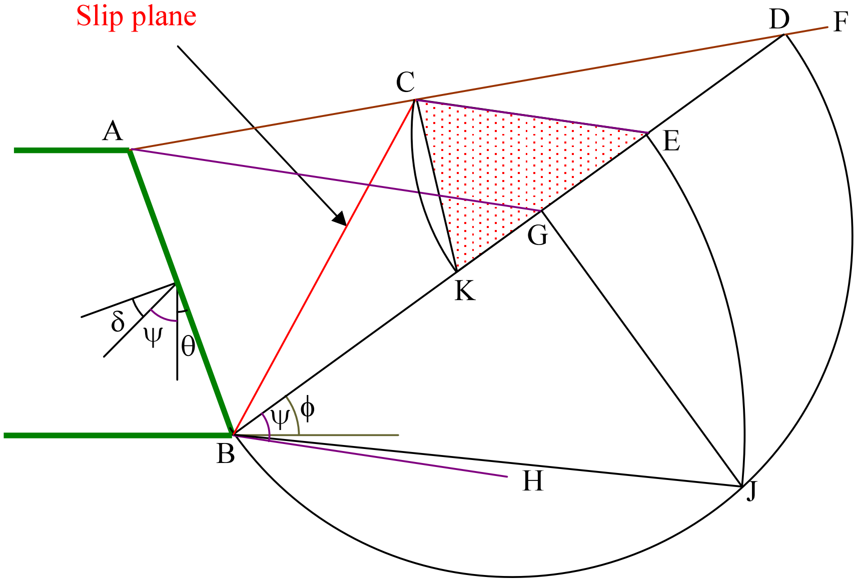


Solution:

$\psi = 90 - \delta - \theta$

$= 90 - 2/3 \times 36 - 20$

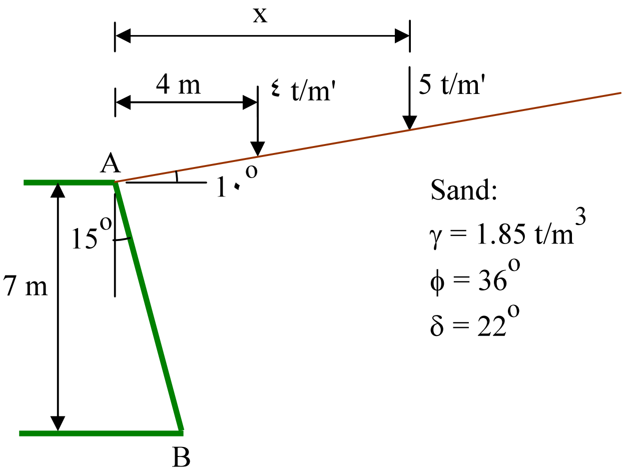
$= 46^{o}$



$P_a = \gamma \times \Delta\, KEC$

$= 1.85 \times 0.5 \times 6.6 \times 4.75 = 29$ t/m'

(9) For the shown wall and backfill, use Culmann's method to determine the value and point of application of active earth pressure on the wall. Find the effect of a line load 4 t/m', located 4 m far from point A, on the active earth pressure value. Find the distance (x) so that the line load will not affect the active earth pressure on the wall.



Solution:

$\psi = 90 - \delta - \theta$

$\quad = 90 - 22 - 15 = 53^{o}$

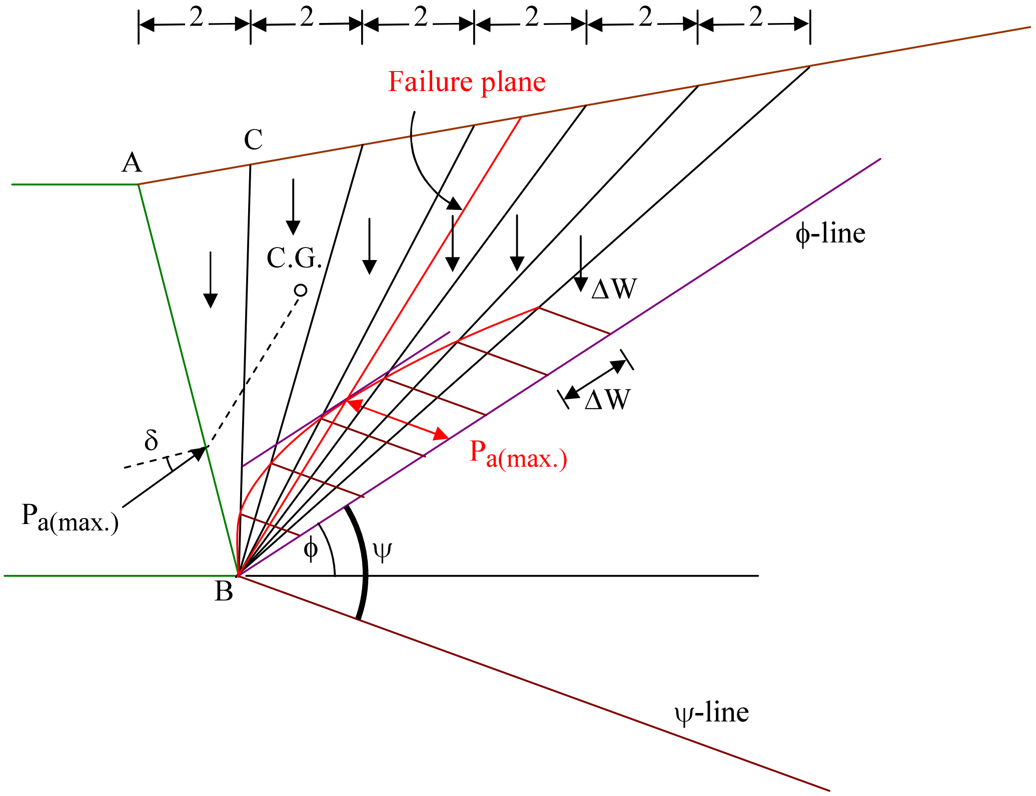
Divide backfill into wedges of equal areas as shown.

AC = 2 / cos 10 = 2.03 m
AB = 7 / cos 15 = 7.25 m

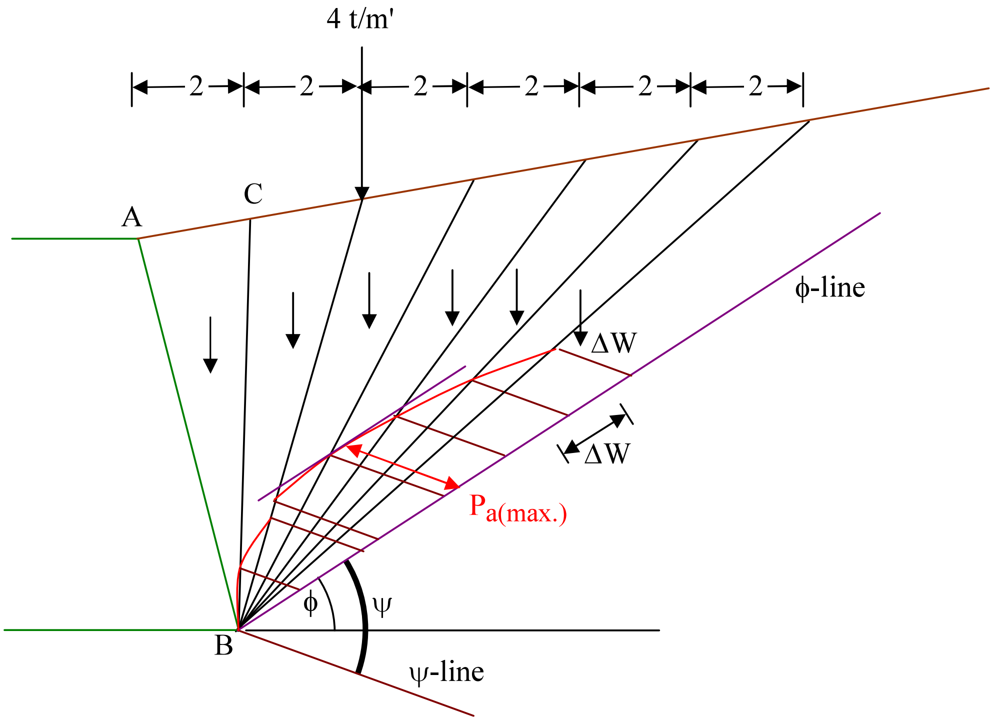
Length of perpendicular from B to AC = 7.25 sin 85 = 7.22 m

Area Δ ABC = 0.5 x 2.03 x 7.22 = 7.33 m^2
ΔW = 7.33 x 1.8 = 13.2 t/m'

- The graphical solution is as shown. Forces are taken on the ϕ-line to scale 1 cm = 10 t/m'
- The value of $P_{a\ (max.)}$ is found graphically to be 20.04 t/m'.
- The location of the failure plane is determined. The C.G. of the failure wedge is determined.
- The point of application is obtained as shown.



To determine the effect of line load, its value is added to the weight of soil wedges as shown. The corresponding new Culmann's line is drawn. In this case the value of $P_{a(max.)}$ is found graphically to be 21.91 t/m'.

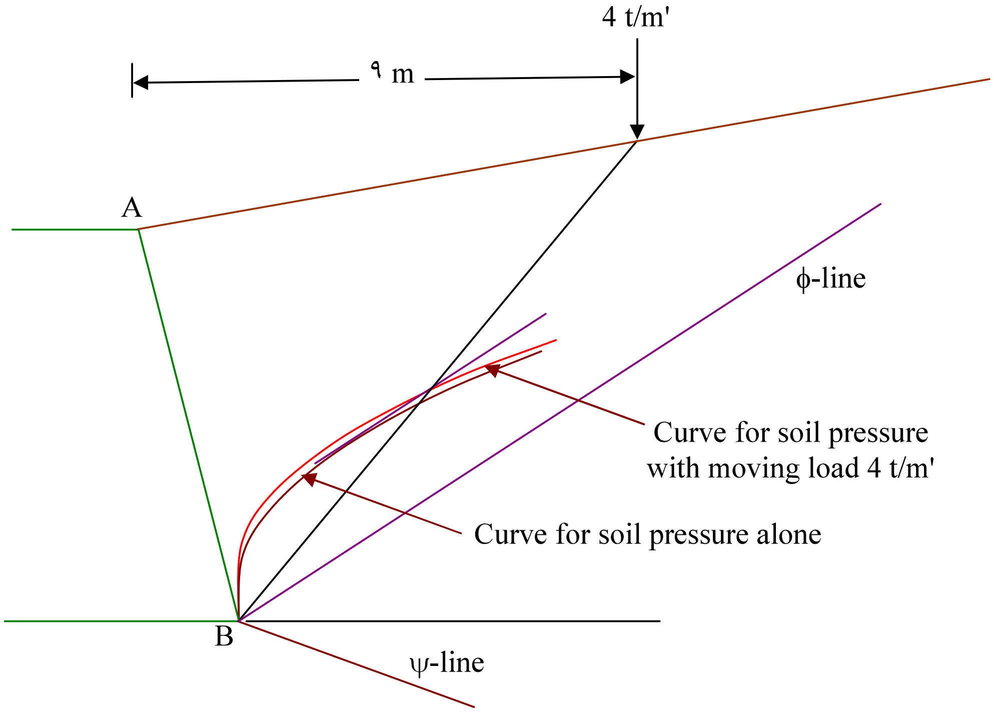


To find the distance (x) beyond which the line load has no effect on the earth pressure:

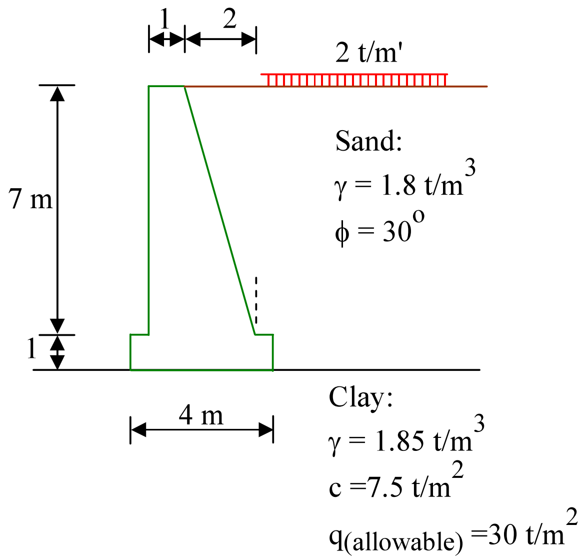
(a) Both original (الاصلى) and new Culmann's lines are drawn.

(b) Plot the tangent to the original line parallel to the f-line. It intersects the new Culmann's line at point J.

(c) Join line BJ and extend it to intersect the ground surface at point K. The horizontal projection of distance AK is found graphically to be 9 m. K is the point beyond which the line load has no effect on the earth pressure

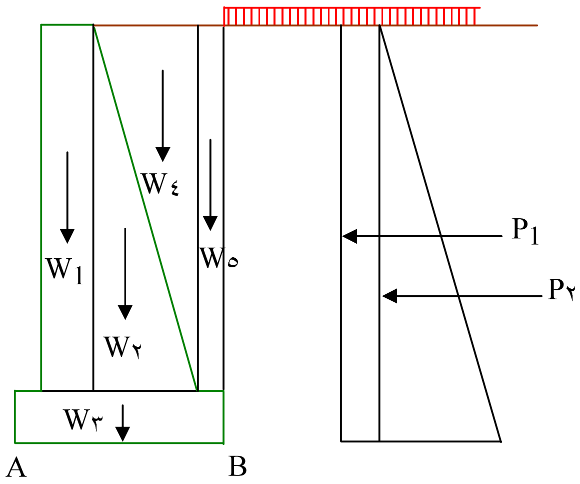


(10) Check the stability of the shown wall. All data are as shown.



Solution:

Case (i): Surcharge is acting outside wall base.



Calculate weights:

$W_1 = 11$ t/m'

$W_2 = 11$ t/m'

$W_3 = 8.8$ t/m'

$W_4 = 9$ t/m'

$W_5 = 4.5$ t/m'

$N = 44.3$ t/m'

Calculate earth pressure:

$P_1 = 4$ t/m'

$P_2 = 10.8$ t/m'

- Check sliding:

Sliding force = 4 + 10.8 = 14.8 t/m'

Force resisting sliding = (2/3) x 7.5 x 4 = 20 t/m'

$F.S._{sliding} = 20 / 14.8 = 1.35$ (< 1.5 unsafe)

Use key with height 0.5 m:

$p_{p1} = (44.3 + 4) \text{ x } 1 + 2 \text{ x } 5\ 41 = 21.075 \text{ t/m}^2$

$p_{p2} = 21.075 + 1.85 \text{ x}.5 \text{ x } 1 = 21.075 + 0.925 = 22 \text{ t/m}^2$

$\Sigma P_p = 0.5\ [\{(21.075\ + 22) + 2\} \text{ x } .5] = 5.38 \text{ t/m'}$

$S = 20 + 5.38 = 25.38 \text{ t/m'}$

$F^{\cdot}S^{\cdot}{}_{\text{sliding}} = 25.38 / 14.8 = 1.72$ (O.K.)

- Check overturning :

$M_O = 4 \text{ x } 3 + 10.8 \text{ x } 2 = 33.6 \text{ t.m/m'}$

M (stability) = Moment of weights W1 to W5 about point A = 94.8 t.m/m'

$F^{\cdot}S^{\cdot}{}_{\text{overturning}} = 94.8 / 33.6 = 2.82$ (O.K.)

- Check tension at base:

M_B = Moment of weights W1 to W5, P1 and P2 about point B

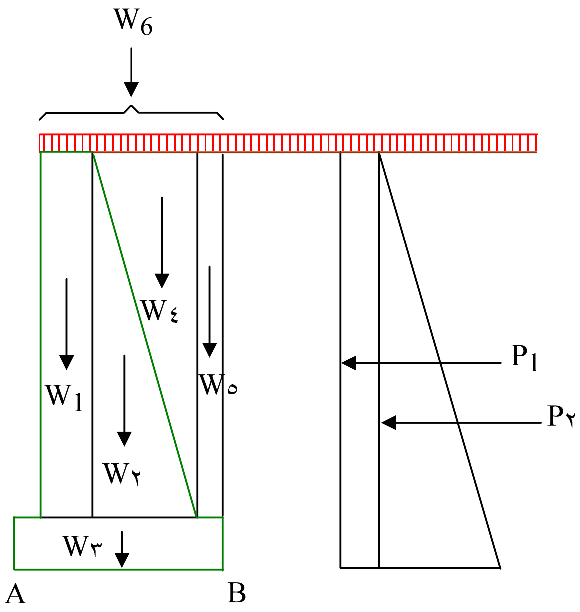
= 115.99 t.m/m'

$e_B = 2.618$ m

e = (2.618 - 4/2) = 0.618 m

$$f_{1,2} = \frac{-44.3}{4}\left(1 + \frac{6 \times 0.618}{4}\right) = -21.4, -0.9 \text{ t/m}^2$$

No tension will occur on base. However, the ratio between f_1 and f_2 is > 2. It may be more suitable to increase the width of wall base to reduce this ratio, depending on the settlement of the wall.



- Check maximum compression at base:

$W6 = 8$ t/m'

$N = 44.3 + 8 = 52.3$ t/m'

MB = Moment of weights W1 to W6, P1 and P2 about point B

= 131.99 t.m/m'

eB = 2.523 m

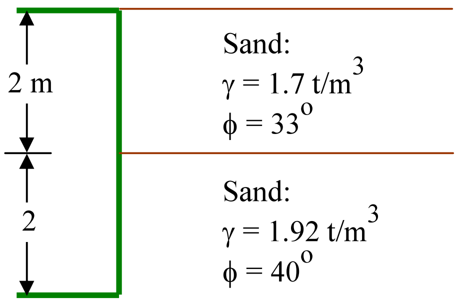
e = (2.523 – 4 / 2) = 0.523 m

$$f_{1,2} = \frac{-52.3}{4}\left(1 + \frac{6 \times 0.523}{4}\right) = -23.3, -2.8 \ \text{t/m}^2 \ (< 30 \therefore \text{O.K.})$$

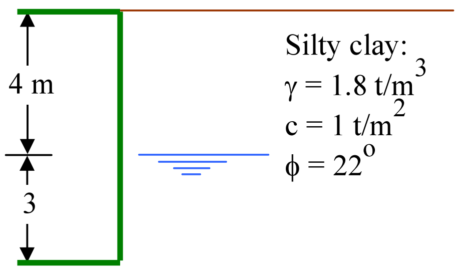
Therefore, the wall is safe with a key at the base. However, if the differential settlement between points A and B is unacceptable (غير مقبول), the width of the wall base should be increase so that the ratio f_1 to $f_٢$ will be within permissible limits (حدود مسموح بها).

13.10 Problems

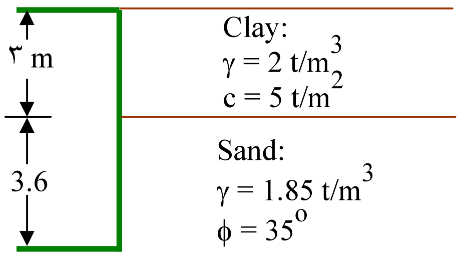
(1) For the shown walls and soil formations, determine the distributions of lateral earth pressure. Find the magnitude and point of application of the resultant earth pressure force per unit length of the wall. Assume that the wall faces are smooth, and apply Rankine's method.



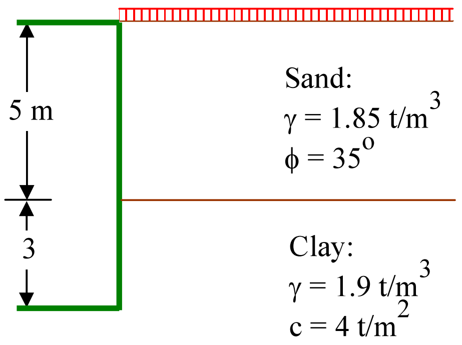
(a)



(b)

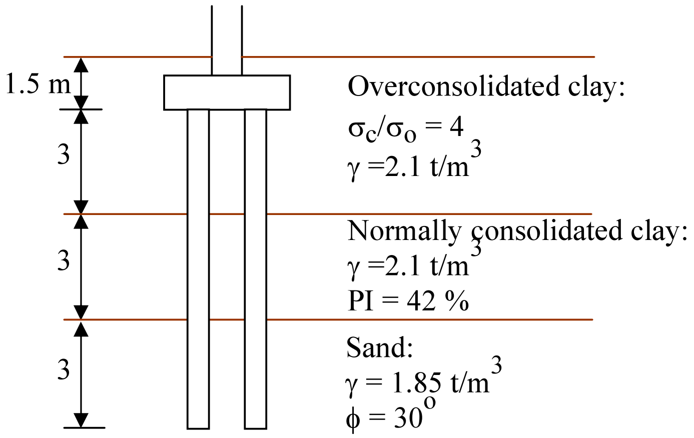


(c)

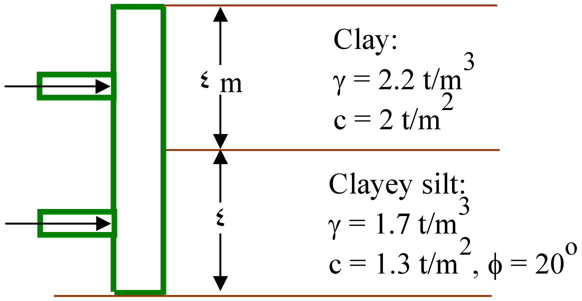


(d)

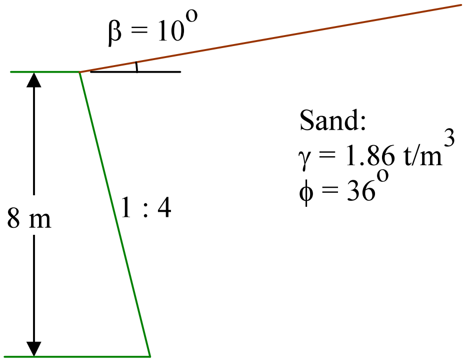
(2) Draw the at rest earth pressure distribution on the shown pile (خازوق).



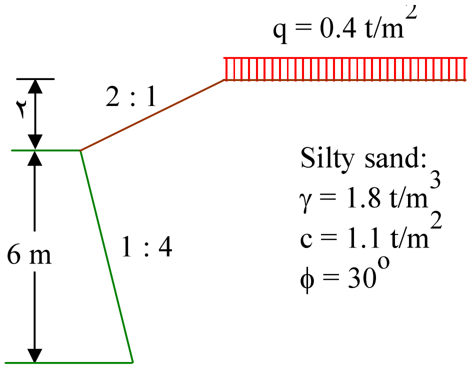
(3) Draw the passive earth pressure distribution on the shown structure.



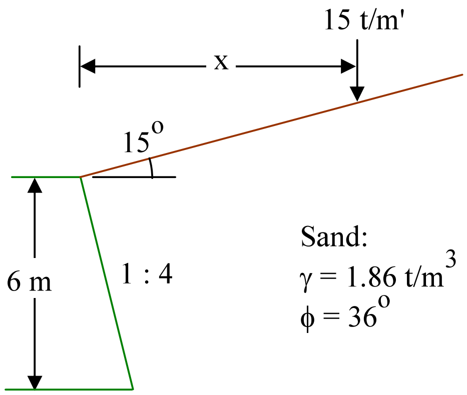
(4) The shown is a retaining wall with smooth back. Find the magnitude and point of application of the active earth pressure on the wall. Assuming that the wall is rough with $\delta = 3/4\ \phi$, use Rebhann's graphical method to determine the magnitude of the active earth pressure on the wall.



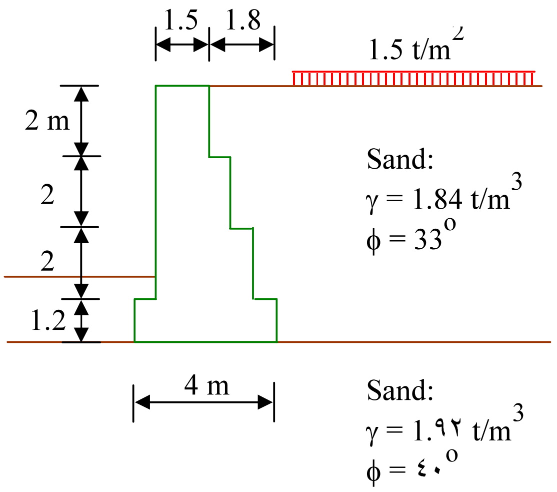
(5) Use Coulomb's graphical method to find the magnitude of the total earth pressure on the shown retaining wall.



(6) Use Culmann's graphical method to find the magnitude of the total earth pressure on the shown retaining wall. Find the distance (x) beyond which the shown concentrated load will not affect the earth pressure on the wall.



(7) Check the stability of the shown retaining wall. The allowable compressive stress on soil is 4 kg/cm^2.



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